CDF note 8963

Measurement of the Charge Asymmetry in Top Pair Production

The CDF Collaboration
URL http://www-cdf.fnal.gov
(Dated: August 17, 2007)

We present a measurement of the charge asymmetry in top pair production using an integrated luminosity of 1.7 fb^{-1} collected during the years 2003 to 2007 with the CDF detector. Top pair candidates with a signature of a charged lepton, missing transverse momentum and at least four jets are selected. In order to measure the charge asymmetry the rapidity difference between the semileptonically decaying and the hadronically decaying top quark multiplied with the charge of the charged lepton is used. A measurement of the inclusive asymmetry, yielding $A^{\Delta y Q_l} = 0.28 \pm 0.13 \pm 0.05$, as well as of the asymmetry in events with exactly 4 or at least 5 reconstructed jets is performed. The measured asymmetries are higher than the asymmetries predicted by the next-to-leading order Monte Carlo generator MC@NLO, but within the uncertainties the measured values are consistent with the NLO predictions.

Preliminary Results for Summer 2007 Conferences

I. INTRODUCTION

In 1995 the top quark was discovered at the Tevatron proton-antiproton collider at Fermilab by the CDF and DØ collaborations [1]. It is the most massive known elementary particle and its mass is currently known with a precision of about 1.1% [2]. However, the measurements of other top quark properties are still statistically limited, so the question remains whether the standard model successfully predicts these properties.

At the Tevatron collider, with a center-of-mass energy of $\sqrt{s} = 1.96$ TeV most top quarks are pair-produced via the strong interaction. The quark annihilation process is expected to contribute with 85%, while the gluon-gluon fusion process is expected to contribute with 15%. Due to the large mass of the top quark, top production is an ideal testing ground to study effects predicted by QCD. Next-to-leading order (NLO) calculations [3, 4] predict in $p\bar{p} \to t\bar{t}X$ reactions a charge asymmetry of the top and anti-top production arising from the interference of initial state gluon radiation (ISR) and final state gluon radiation (FSR) on the one hand, and interference of born and box diagram on the other hand (see figure 1). Only quark anti-quark annihilation $q\bar{q} \to t\bar{t}X$ and heavy flavor excitation $qg \to qt\bar{t}$, which can be neglected at Tevatron energies, are charge asymmetric, while gluon fusion is charge symmetric.

The asymmetry occurs in the variable $\cos\alpha$, where α is the angle of the top quark in the rest frame of the incoming partons, and is defined as [4]:

$$A(\cos\alpha) = \frac{N_t(\cos\alpha) - N_{\bar{t}}(\cos\alpha)}{N_t(\cos\alpha) + N_{\bar{t}}(\cos\alpha)} = \frac{N_t(\cos\alpha) - N_t(-\cos\alpha)}{N_t(\cos\alpha) + N_t(-\cos\alpha)} \tag{1}$$

Because the charge conjugation symmetry holds for the strong interaction, implying $\sigma_{q\bar{q}\to t\bar{t}}(\alpha) = \sigma_{\bar{q}q\to \bar{t}t}(180^\circ - \alpha)$, this charge asymmetry can be interpreted as forward-backward asymmetry. Calculating the total forward-backward asymmetry

$$A^{\cos\alpha} = \frac{N_t(\cos\alpha \geq 0) - N_t(\cos\alpha < 0)}{N_t(\cos\alpha \geq 0) + N_t(\cos\alpha < 0)} \tag{2}$$

in $p\bar{p} \to t\bar{t}$ reactions at Tevatron energies with respect to the total NLO top pair production cross section leads to a predicted asymmetry of $(4-6)$%. A significantly larger forward-backward asymmetry would indicate new physics, like a Z' [5], decaying dominantly in $t\bar{t}$.

In the standard model, according to [4] the interference of the box and LO amplitudes leads to a positive asymmetry while the interference between the ISR and FSR amplitudes yields a negative value. Since the absolute value of the former one is always larger than that of the latter one a slightly positive asymmetry is obtained in total. LO $t\bar{t}+$jet calculations yield an asymmetry of about $-(9-10)$%, while the recent NLO $t\bar{t}+$jet calculation [6] yields $-(0-2)$%. Here a factor 1.3 is applied to the values in [6] to account for the difference in the asymmetry calculated in the $p\bar{p}$ frame or in the parton-parton center of mass frame. Studying the asymmetry separately for events with either none ($t\bar{t}$) or one additional hard gluon ($t\bar{t}g$), information about the different contributions to A can be gained. Experimentally no clear distinction between those event classes is possible. However, the number of additional hard objects is reflected in the number of reconstructed jets and thus well suited to study both contributions to A. In a Monte Carlo study we observe indeed a strong dependence of the charge asymmetry on the number N_{jets} of reconstructed jets as presented in figure 2 a). For events with low N_{jets} the visible charge asymmetry corresponds well to the asymmetry of the $t\bar{t}$ contribution, while the asymmetry for events with large N_{jets} corresponds well to the asymmetry of the $t\bar{t}g$ contribution.

In order to study the charge asymmetry in top pair production we select top anti-top events in the lepton+jets channel, reconstruct the four-vectors of the top quarks for each $t\bar{t}$ candidate. Unfortunately, the variable $\cos\alpha$ is experimentally not accessible and in NLO Monte Carlos like MC@NLO [7] this variable is not unambiguously defined. Therefore, we use in our analysis not $\cos\alpha$ itself but the rapidity difference of the top quark y_t and the anti-top quark $y_{\bar{t}}$. $y_t - y_{\bar{t}}$ is lorentz-invariant and in LO directly related to $\cos\alpha$ [8]:

$$y_t - y_{\bar{t}} = 2 \cdot \operatorname{atanh}\left(\frac{\cos\alpha}{\sqrt{1 + \frac{4m_t^2}{\hat{s} - 4m_t^2}}}\right) \tag{3}$$

Here, $\hat{s}$ indicates the center of mass energy of the initial partons and m_t the top mass. Experimentally, $y_t - y_{\bar{t}}$ is calculated by the rapidity difference of the semileptonically ($y_{t_{lep}}$) and the hadronically ($y_{t_{had}}$) decaying top quark multiplied by the charge of the lepton (e or μ) from the semileptonic top decay (see figure 3 a)):

$$\Delta y \cdot Q_l = (y_{t_{lep}} - y_{t_{had}}) \cdot Q_l = y_t - y_{\bar{t}}. \tag{4}$$

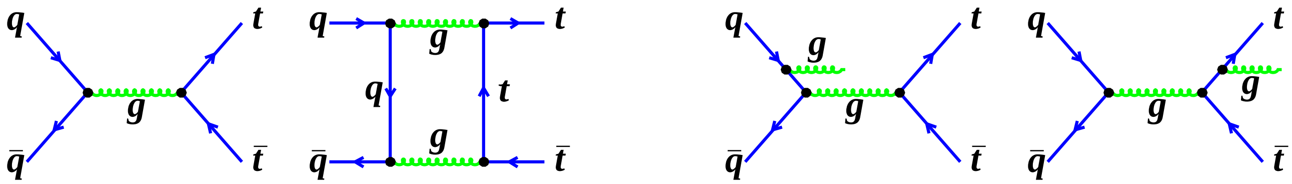

FIG. 1: *Quark-antiquark annihilation diagrams.*

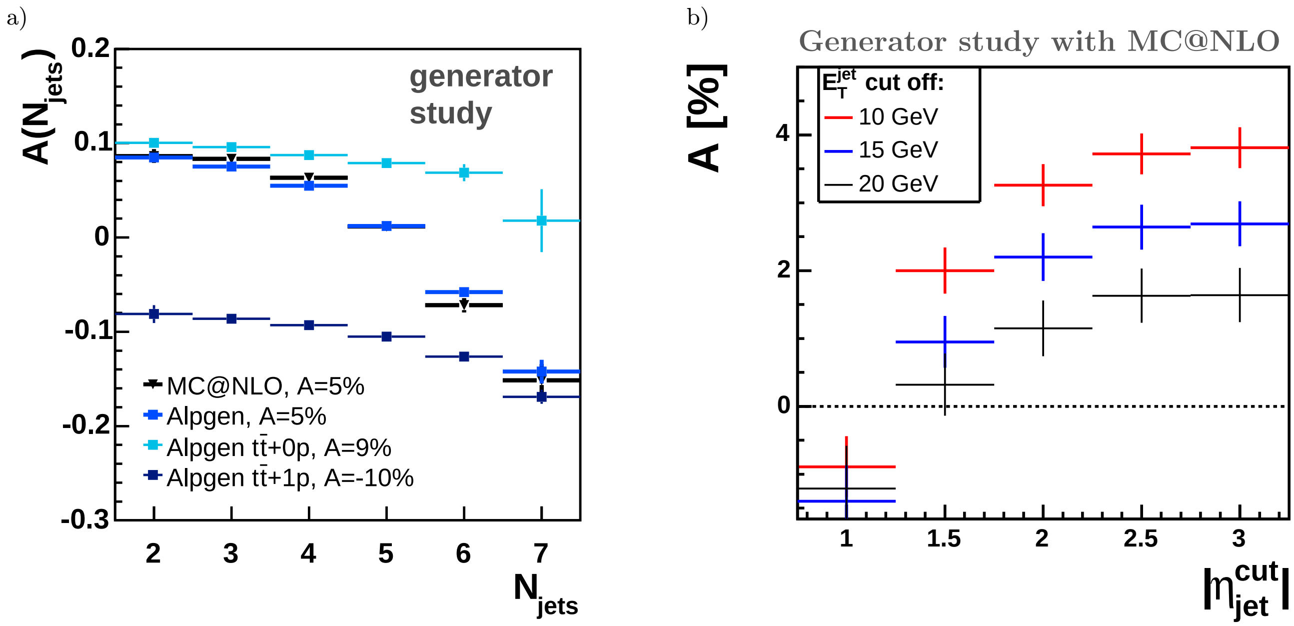

FIG. 2: *a) Asymmetry as a function on the number of reconstructed jets (default jet cuts: $|\eta| < 2.0$, $E_T > 20$ GeV). b) Asymmetry as function of different pseudorapidity cuts on the jets and separately for different cuts on the transverse energy of the jets.*

Because no sign inversion occurs between $\cos\alpha$ and $\Delta y \cdot Q_l = y_t - y_{\bar{t}}$ for all values of $\sqrt{\hat{s}}$ at LO, the measurement of the total asymmetry $A^{\Delta y \cdot Q_l}$ in the observable $\Delta y \cdot Q_l$ represents the total asymmetry $A^{\cos\alpha}$ in $\cos\alpha$ reasonably well. However, the inclusive asymmetry in $\Delta y \cdot Q_l$ is not exactly equal to the asymmetry in $\cos\alpha$, so we state for the prediction of $A^{\Delta y Q_l}$ $(4-7)\%$. In case of MC@NLO with CTEQ5M as parametrization of the parton distribution functions (PDF) an asymmetry $A^{\Delta y Q_l}$ of 5% is obtained, see figure 3b).

The event selection, smearing and resolutions effects as well as background distort the true $(\Delta y \cdot Q_l)_{gen}$ distribution. As investigated in detail in [9], in particular the cuts on our selected jets reduce the charge asymmetry in the visible range. This is pictured in figure 2 b). The harder the cuts on the selected jets, the larger is the fraction of the $t\bar{t}g$ component and thus the smaller is the visible charge asymmetry.

In case of the inclusive asymmetry, the reconstructed asymmetry A^{rec}:

$$A^{rec} = \frac{N^{rec}_{pos} - N^{rec}_{neg}}{N^{rec}_{pos} + N^{rec}_{neg}} \tag{5}$$

obtained from the reconstructed $\Delta y \cdot Q_l$ distribution is corrected to obtain the "true" asymmetry $A^{\Delta y \cdot Q_l} \approx A^{\cos\alpha}$. For that, the background is subtracted first and then the background subtracted number of events for positive $N^{bg\,sub}_{pos}$ and negative $N^{bg\,sub}_{neg}$ values of $\Delta y \cdot Q_l$ are corrected for smearing and resolutions effects as well as for event selection efficiency. In case of the reconstructed asymmetry in events with exactly 4 or at least 5 jets respectively only the background is subtracted and then a comparison with the prediction of MC@NLO and the LO+PS Monte Carlos Pythia and Herwig is performed.

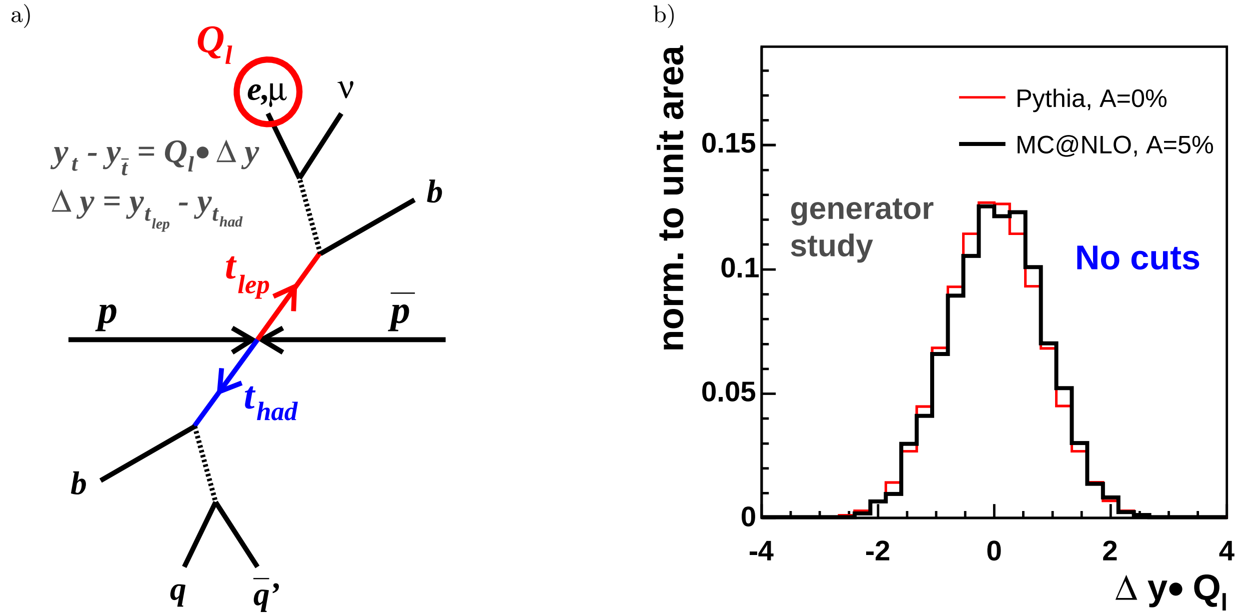

FIG. 3: *Sensitive variable $\Delta y \cdot Q_l$. a) Definition of $\Delta y \cdot Q_l$. b) Distribution of $\Delta y \cdot Q_l$ without applying any cuts for the Monte Carlo generators* PYTHIA *and* MC@NLO.

The charge asymmetry has recently been measured by the DØ collaboration to be $(12 \pm 8 \pm 1)\%$ in the experimentally visible phase space [10], which is consistent with the prediction by MC@NLO of $(0.8 \pm 0.2 \pm 1.0)\%$. The DØ analysis makes also use of $y_t - y_{\bar{t}}$ as sensitive variable, while a second analysis [11] of the CDF collaboration uses $(-Q_l) \cdot \cos\theta_{t_{had}}$, where $\theta_{t_{had}}$ is the polar angle of the hadronically decaying top quark, as sensitive variable. The latter analysis thus measures the asymmetry in the lab frame in contrast to the analysis presented in this note and the analysis of DØ which are both performed in the parton rest frame. The asymmetry in the lab frame is reduced by about a factor of 1.3 compared to the asymmetry measured in the parton rest frame.

II. THE CDF II EXPERIMENT

A detailed description of the Collider Detector at Fermilab (CDF) can be found elsewhere [12]. A coordinate system with the z axis along the proton beam, azimuthal angle ϕ, and polar angle θ is used. The pseudorapidity is defined as $\eta = -\ln\tan(\theta/2)$. The transverse energy of a particle is defined as $E_T = E\sin\theta$. The primary detector components relevant to this analysis are those that measure the jet, electron, and muon energies and directions.

An open cell drift chamber, the Central Outer Tracker (COT), and a silicon tracking system are used to measure the momenta of charged particles. The CDF II silicon tracker consists of three sub detectors: (1) a layer of single-sided radiation resistant silicon microstrip detectors glued on the beam pipe, (2) a five layer double-sided silicon microstrip detector (SVXII), and (3) additional Intermediate Silicon Layers located at radii between 19 and 30 cm provide good linking between the track segments in the COT and the SVXII. In the analysis presented in this paper the silicon tracker is particularly important to identify jets originating from b quarks by reconstructing secondary vertices. The tracking chambers are all located within a 1.4 T axial magnetic field. The pseudorapidity coverage of the COT is $|\eta| < 1.1$, while the silicon system reaches up the $|\eta| < 2.0$. All electromagnetic and hadronic calorimeters at CDF are used to measure the jets energy. In this analysis jets are reconstructed in the pseudorapidity range of $|\eta| < 2.0$. The Central Electromagnetic and Central Hadronic Calorimeter with an angular coverage of $|\eta| < 1.1$ are used to identify electron candidates. The Central Muon System, Central Muon uPgrade and the Central Muon eXtension, with a total coverage of $|\eta| < 1.0$ are used to identify muons.

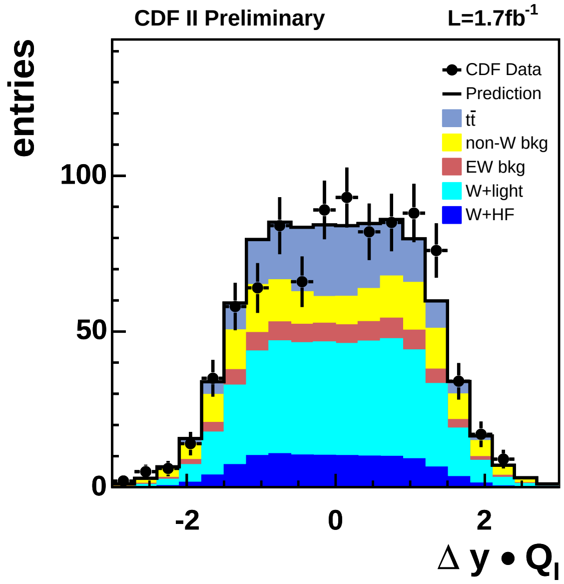

FIG. 4: *Modeling of the background. Shown is the reconstructed $\Delta y \cdot Q_l$ distribution in the background dominated sample.*

III. EVENT SELECTION OF $t\bar{t}$ CANDIDATES

We select $t\bar{t}$ candidate events, where one top quark decays semileptonically, $t \to b\ell\nu$, and the second top quark decays hadronically, $\bar{t} \to \bar{b}q\bar{q}'$. The charged lepton is either identified as an electron or muon candidate. The branching ratio of this lepton + jets channel is about 30%.

Top quark candidates in the lepton + jets channel are selected by requiring evidence for a leptonic W decay: (a) missing transverse energy $\not\!\!E_T > 20$ GeV from the neutrino and (b) exactly one well isolated central electron candidate with $E_T > 20\,\mathrm{GeV}$ and $|\eta| < 1.1$, or exactly one well isolated central muon candidate with $p_\mathrm{T} > 20\,\mathrm{GeV}/c$ and $|\eta| < 1.0$. An electron or muon candidate is considered isolated if the non-lepton E_T in an $\eta - \phi$ cone of radius 0.4 centered around the lepton is less than 10% of the lepton E_T or p_T, respectively. Jets are reconstructed using a fixed cone of radius $\Delta R = \sqrt{\Delta\phi^2 + \Delta\eta^2} = 0.4$. We count jets with $E_T > 20$ GeV and with a pseudorapidity of $|\eta| < 2.0$. Only events with at least four jets are accepted. Because in $t\bar{t}$ events two b jets should exist, we require, that at least one of these jets must be likely to originate from a b quark (b-tag) by requiring a displaced secondary vertex within the jet as measured using silicon tracker information. Altogether we select 448 $t\bar{t}$ candidate events.

IV. SIGNAL SIMULATION AND BACKGROUND ESTIMATION

In this analysis Monte Carlo simulations are used to determine the efficiencies and the resolution due to the reconstruction of top-pair signal events as well as to compare data with. All generated events are passed through the CDF detector simulation. Afterwards the same reconstruction as for real data is applied.

$t\bar{t}$ signal samples are generated with the leading order plus parton shower Monte Carlo generators PYTHIA [13] and HERWIG [14], with the matrix element generator ALPGEN [15], where the showering is performed with Pythia, as well as with NLO Monte Carlo generator MC@NLO [7] using a top mass of $m_t = 175$ GeV/c^2.

The selected $t\bar{t}$ candidates in data still contain some background contamination. We observe 448 events with a background estimation of 76.24 ± 20.44. One source of background events are W-boson plus jets events. Here two different types of W-production have to be distinguished. The first category are W events, where the jets originate from light quarks. In this case one jet is misidentified as a b-quark jet (mistags). The second category are W events with one or more jets originating from a c- or b-quark (W + heavy flavor events). A further source of background are QCD processes, where one jet fakes the charged lepton and another jet is either truly identified as a b-quark jet or is misidentified as a b-quark jet. This background is called QCD background or non-W background. In addition electroweak processes, like di-boson (ZW, WW, ZZ) and single top production contribute to the background. However, the fraction of these backgrounds is rather small and can be determined from the Monte-Carlo simulation.

About 5% of all real $t\bar{t}$ events, selected with our cuts, do not belong to the $t\bar{t}$ lepton+jets channel, but to either the $t\bar{t}$ all hadronic or dilepton channel. These events are treated as background events, leading to a total background estimation of 96.10 events.

The modeling of the background has been checked carefully using a background dominated sample. This sample is selected in the same way as our $t\bar{t}$ candidates except of vetoing jets to be tagged as a b-jet. As presented in figure 4 exemplary for our sensitive variable, the shape of the background is modeled well.

V. FULL RECONSTRUCTION OF TOP ANTI-TOP PAIRS

Due to the incomplete measurement of the neutrino four momentum and several possibilities to assign the jets to the decay products of the top quarks the reconstruction of $t\bar{t}$ pairs has to handle with several possible event hypotheses. For the reconstruction of $t\bar{t}$ events the selected jets and the missing transverse energy are corrected to parton level.

Both top quarks are reconstructed from the measured four momenta of their decay particles. Since the neutrino does not interact with the detector, it appears only in the missing transverse energy. Thus only the x and y component of the neutrino momentum are known. The missing z component is calculated using a W mass constraint on the W boson decay. This treatment leads to a quadratic equation for p_z of the neutrino. In 70% of all cases this results in an ambiguity of two real solutions for $p_{z,\nu}$, which have both to be taken into account. In the remaining 30% of events the solution of the quadratic equation becomes complex. In these cases we vary the x and y component starting from the measured values until the imaginary part of the p_z solution vanishes. Thus this treatment leads to one solution for the z component of the neutrino momentum.

We consider all possibilities to assign the jets in the event to the two b quarks and the two light quarks from the $t\bar{t}$ decay. It should also be mentioned that we take all jets of the event into account and not only the four leading jets. This procedure leads to a multiplicity of possibilities for the reconstruction of the event. Due to the N_ν (2 or 1) solutions for the z-component of the momentum of the neutrino and the ways to assign the selected jets to the four jets in the $t\bar{t}$ decay, $N_\nu \cdot N_{jets} \cdot (N_{jets}-1) \cdot (N_{jets}-2) \cdot (N_{jets}-3)/2$ hypotheses for the complete kinematic reconstruction of a $t\bar{t}$ event candidate are obtained.

In order to choose the best event interpretation, a quantity Ψ is determined for each hypothesis, which gives a quantitative estimate how well the hypothesis matches the $t\bar{t}$ pair assumption. Ψ is defined by:

$$\Psi = P_\nu \cdot P_{\mathrm{b-light}} \cdot \chi^2 \tag{6}$$

The quantities entering the computation of Ψ are:

1. $P_\nu = 0.29$ ($P_\nu = 0.71$) for solution with smaller (larger) $|p_{z,\nu}|$ (in case of two real solutions)
2. $P_{\mathrm{b-light}}$: A measure for the light quark likeness of the jets assigned as b jets.
3. χ^2: Constraints on the mass of the hadronically decaying W boson, on the mass difference between both reconstructed top masses (two particles with the same mass), and on the transverse energy of the two top quarks

Here, P_ν can be interpreted as the probability for the chosen neutrino solution to be the wrong one.
χ^2 is defined via:

$$\chi^2 = \frac{(m_{W\to jj} - M_{W\to jj})^2}{\sigma^2_{M_{W\to jj}}} + \frac{(m_{top\to b\ell\nu} - m_{top\to bjj})^2}{\sigma^2_{\Delta M_t}} + \frac{(P_{energy} - \alpha)^2}{\sigma^2_{P_{energy}}} \tag{7}$$

In the first term $m_{W\to jj}$ is the reconstructed mass of the hadronically decaying W boson, which should be equal to the mean value $M_{W\to jj}$ of the $m_{W\to jj}$ distribution within the resolution $\sigma_{M_{W\to jj}}$. In the second term ΔM_t is the difference between the reconstructed mass of the semileptonically decaying top $m_{top\to b\ell\nu}$ and the mass of the hadronically decaying top quark $m_{top\to bjj}$. Since the two top quarks are identical particles, the mass difference of both reconstructed top quarks is assumed to be zero, within the uncertainty $\sigma_{\Delta M_t}$. P_{energy} is the fraction of the sum of transverse energies of the two top quarks and the total transverse energy of the event including missing transverse energy. The values for $M_{W\to jj}$, $\sigma_{M_{W\to jj}}$, $\sigma_{\Delta M_t}$, and α are obtained from MC studies.

$P_{b-light}$ is a measure for the light-quark likeness of the jets assigned as b jets and is defined as:

$$P_{b-light} = (JP_{top\to b\ell\nu} + (1 - R'_{top\to b\ell\nu})) \cdot (JP_{top\to bjj} + (1 - R'_{top\to bjj})) \tag{8}$$

	Asymmetry [%]		
	$N_{jets} \geq 4$	$N_{jets} = 4$	$N_{jets} \geq 5$
Wbb	-5.0 ± 0.4	-5.3 ± 0.4	-3.6 ± 0.9
Wcc	-3.8 ± 0.2	-3.6 ± 0.2	-5.0 ± 0.4
Wljets	-0.6 ± 1.5	-1.0 ± 1.7	1.0 ± 3.4
EW	-1.5 ± 4.4	-1.9 ± 5.5	0.4 ± 24.1
QCD	0.6 ± 1.0	0.7 ± 1.1	0.5 ± 1.8

TABLE I: *Asymmetries of the different background components in our selected sample.*

Here $JP_{top \rightarrow b\, l \nu}$ and $JP_{top \rightarrow b\, jj}$ are the probability of the jet [16] chosen to be the b jet from the semileptonically and hadronically decaying top quark, respectively, to be consistent with a zero lifetime hypothesis, i.e. to be a light quark jet. This probability is calculated from the positive impact parameter in the $r - \phi$-plane of the tracks assigned to the jet. For jets with a well displaced secondary vertex a more accurate b-likeness measure R' is calculated using the output of a neural network b-tagger, while R' is set to zero otherwise. Since $P_{b-light}$ is defined as the probability for the assigned b jets to be light quark jets, we have to use (1-R') instead of R' in equation 8.

Ψ is calculated for each hypothesis in the event and we then choose for each event the hypothesis with the smallest value of Ψ.

VI. RECONSTRUCTED AND BACKGROUND SUBTRACTED ASYMMETRY

The distribution of the reconstructed $\Delta y \cdot Q_l$ using the best event interpretation together with the background estimate is presented in figure 5. In a) the distribution for the inclusive sample with at least four reconstructed jets ($N_{jets} \geq 4$) is presented while in b+c) the distributions for the exclusive jet bins $N_{jets} = 4$ and $N_{jets} \geq 5$ are presented. From these raw data distributions an asymmetry of $A^{rec} = 0.112 \pm 0.047$ is calculated for the inclusive measurement, while for $N_{jets} = 4$ the asymmetry is determined to $A^{rec} = 0.114 \pm 0.054$ and for $N_{jets} \geq 5$ it is computed to $A^{rec} = 0.105 \pm 0.097$.

As indicated in table I the non-top background is almost symmetric. W+jets events (EW process) are slightly asymmetric, while the QCD background is symmetric. Subtracting both top and non-top backgrounds from the number of events N^{rec}_{pos} (N^{rec}_{neg}) with positive (negative) $\Delta y \cdot Q_l$ value yields for the inclusive sample with $N_{jets} \geq 4$ $A^{bg\, sub} = 0.144 \pm 0.067$. In case of $N_{jets} = 4$ an asymmetry of 0.156 ± 0.078 is computed, while an asymmetry of 0.108 ± 0.127 is calculated for $N_{jets} \geq 5$. This result is visualized for the exclusive measurement in figure 6. Here, the asymmetry obtained from the background subtracted $\Delta y \cdot Q_l$ distribution as a function of the number of reconstructed jets N_{jets} is shown together with the predictions of the LO+PS Monte Carlos PYTHIA and HERWIG and the NLO+PS Monte Carlo MC@NLO. The measured asymmetry values are in both bins above the predictions but consistent with the predictions within the statistical uncertainties. According to the ALPGEN Monte Carlo generator the fraction of $t\bar{t}$ ($t\bar{t}g$) events is about 85% (15%) for events with exactly 4 jets, while the fraction of $t\bar{t}$ events is reduced in case of at least five jets to about 47%. We conclude, that there is so far no indication, that either the $t\bar{t}$ interference contribution (LO and box diagram) or the $t\bar{t}g$ interference contribution (ISR and FSR diagrams) alone are responsible for the high asymmetry value seen in data.

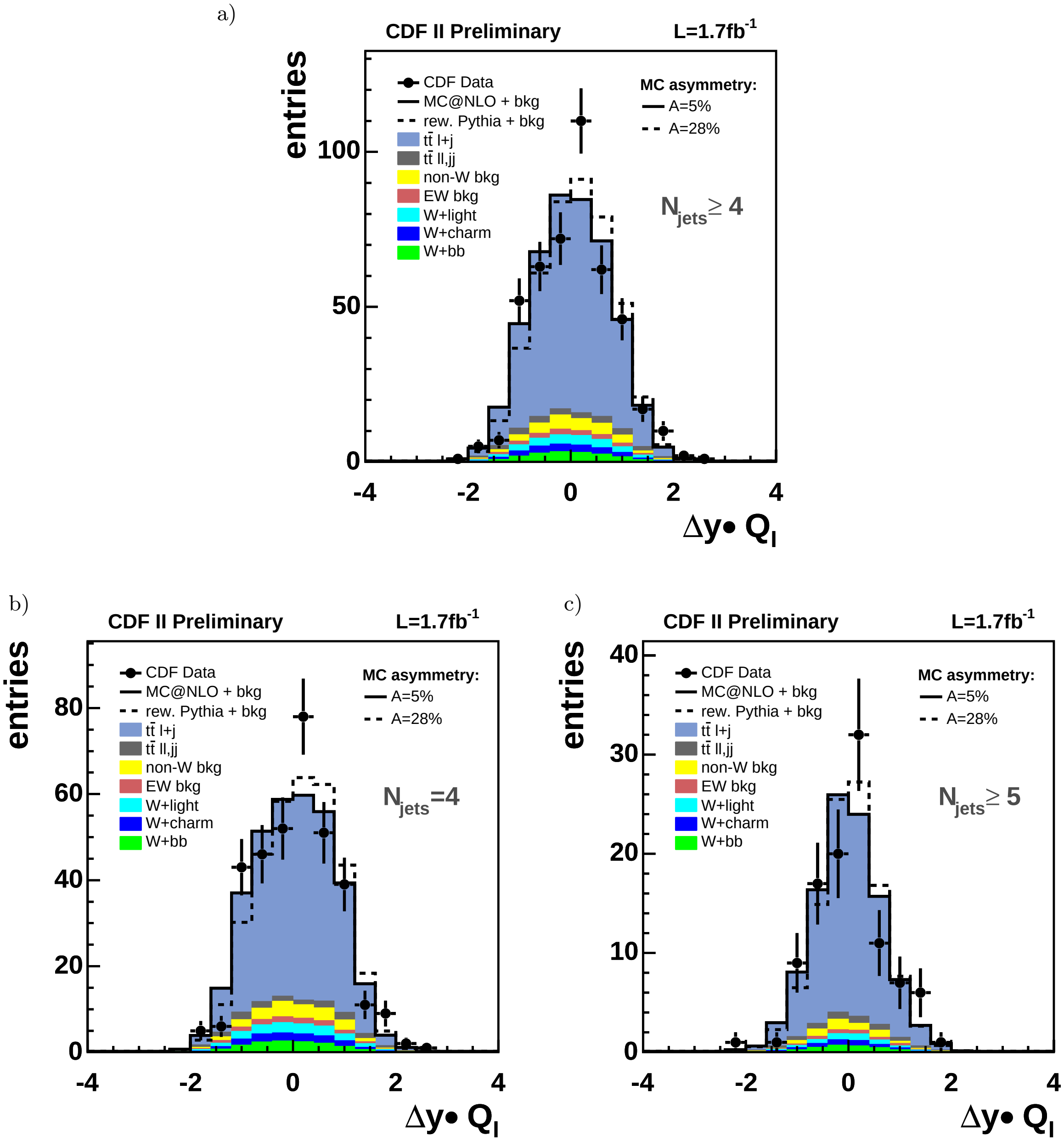

FIG. 5: *Reconstructed $\Delta y \cdot Q_l$ distribution. Shown are the data together with the background estimate, the $t\bar{t}$ MC@NLO prediction and a reweighted Pythia prediction (A=28%). a) Inclusive distribution: $N_{jets} \geq 4$; b) $N_{jets} = 4$; c) $N_{jets} \geq 5$.*

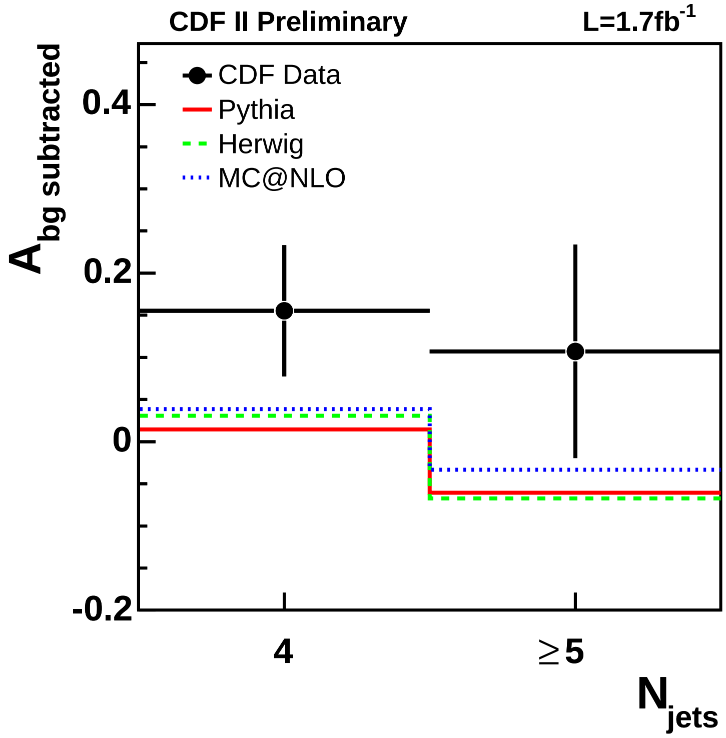

FIG. 6: *Exclusive asymmetry measurement. Shown is the background subtracted asymmetry as a function of the number of reconstructed jets* N_{jets}.

	Pythia
S_{11}	0.736 ± 0.002
S_{21}	0.254 ± 0.002
S_{12}	0.264 ± 0.002
S_{22}	0.746 ± 0.002
ϵ_{11}	1.029 ± 0.003
ϵ_{22}	0.971 ± 0.003

TABLE II: *Smearing matrix elements and efficiencies determined for the correction of the inclusive asymmetry. All values are determined with the* Pythia *Monte Carlo generator.*

VII. EXTRACTION OF THE INCLUSIVE $A^{\Delta y Q_l}$

The background subtracted inclusive asymmetry is corrected for smearing effects due to a non-perfect reconstruction of the top pair kinematic and for the selection efficiency. This correction is performed by inverting a 2x2 matrix.

The reconstructed number of $t\bar{t}$ signal events separately for positive and negative $\Delta y \cdot Q_l$ values is obtained from the theoretical event numbers N_{pos} and N_{neg}, normalized to the total number of background subtracted $t\bar{t}$ events, by accounting for acceptance and migration effects:

$$\begin{pmatrix} N_{neg}^{bg\,sub} \\ N_{pos}^{bg\,sub} \end{pmatrix} = \underbrace{\begin{pmatrix} S_{11} & S_{21} \\ S_{12} & S_{22} \end{pmatrix} \cdot \begin{pmatrix} \epsilon_{11} & 0 \\ 0 & \epsilon_{22} \end{pmatrix}}_{:=R} \cdot \begin{pmatrix} N_{neg} \\ N_{pos} \end{pmatrix} \tag{9}$$

Here S is the smearing matrix, ϵ_{11} (ϵ_{22}) indicates the relative efficiency for selecting events with positive (negative) $(\Delta y \cdot Q_l)_{gen}$. The smearing matrix element S_{ik} gives the probability for an event which was generated in bin i of $(\Delta y \cdot Q_l)_{gen}$ to occur in bin k of the reconstructed $\Delta y \cdot Q_l$ distribution. Because all events of bin i have to occur somewhere in the $\Delta y \cdot Q_l$ distribution, S_{ik} is defined in such a way, that $\sum_k S_{ik} = 1$ holds for all bins i. For the charge asymmetry measurement only two bins are used, namely the bin with negative $(i, k = 1)$ or positive $(i, k = 2)$ values of $(\Delta y \cdot Q_l)_{gen}$ or $\Delta y \cdot Q_l$. The relative efficiencies as well as the smearing matrix elements are given in table II.

With C being R^{-1}, $y^T = (N_{neg}, N_{pos})$ and $x^T = (N_{neg}^{bg\,sub}, N_{pos}^{bg\,sub})$ equation (9) can be rewritten as a linear transformation:

$$y = C \cdot x \tag{10}$$

Computing y according to equation 10, the calculation of the real charge asymmetry yields $A^{cor} = (30.0 \pm 13.3)\%$ for the inclusive measurement.

The extraction method was extensively investigated in pseudo experiments using reweighted Pythia Monte Carlo events with different introduced initial asymmetries. In addition Monte Carlo samples with non-zero initial asymmetries have been used in a further check. Here MC@NLO samples generated with either CTEQ5M (default) or MRST02 as parametrization for the parton distribution functions as well as a $t\bar{t} + 0\,$partons and a $t\bar{t} + 1\,$partons sample, generated with the Monte Carlo Generator ALPGEN, have been utilized.

In all these studies it turned out, that a small linear correction of about 6%, precisely $A = 0.94 \cdot A^{cor}$, is necessary to match the real asymmetry value. Such a correction is understandable, because the 2x2 correction matrix C does slightly depend on the real asymmetry. Although the relative efficiencies and smearing matrix elements do not depend on the real asymmetry for small bin sizes, that is not completely true for the case of only 2 bins. Because the $\Delta y \cdot Q_l$ distribution is different for different initial asymmetries, the integration over these different distributions yields slightly different efficiencies and smearing matrix elements in case of only 2 bins.

Applying the linear correction of about 6%, the result is $A^{\Delta y Q_l} = (28 \pm 13)\%$. The measured asymmetry value is higher than the NLO QCD prediction of $(4-7)\%$ but it is consistent with the NLO prediction within the uncertainties.

VIII. SYSTEMATIC UNCERTAINTIES

The systematic uncertainties caused by the theoretical modeling and the experimental setup are studied performing pseudo experiments. All systematic uncertainties are estimated using Monte Carlo samples with an introduced asymmetry of the size of the measured value, thus with an asymmetry of 28%. The estimated uncertainties are presented in table III.

Source	Uncertainties $\Delta A^{\Delta y Q_l}$
Monte Carlo generator (Herwig)	0.019
Parton distribution function	0.011
ISR / FSR	0.012
Jet energy scale	0.019
BG, shape	0.037
Number of z-vertices	0.011
Fake charge rate	0.013
Total	**0.051**

TABLE III: *Summary of systematic uncertainties for the inclusive asymmetry measurement (determined at A=28%). The total uncertainty is calculated by adding all single uncertainties in quadrature.*

We estimate the possible bias from Monte Carlo modeling of $t\bar{t}$ events by comparing HERWIG and PYTHIA event generators. The influence of initial and final state radiation is estimated by comparing templates from Pythia Monte Carlo simulations in which the parameters for gluon radiation are varied to produce either less or more initial or final-state radiation compared to the standard setup. To estimate the contribution to the total systematic uncertainty that arises from the uncertainty on the PDF, we compare the default PDF to other PDFs.

The uncertainty due to the jet energy scale is quantified by varying that correction within one standard deviation in both the negative and positive direction. No systematic uncertainty on the background normalization is estimated because the uncertainty of the background normalization is already taken into account in the statistical uncertainty. The uncertainty due to the background shape uncertainty is estimated by varying separately the background estimate for each non-$t\bar{t}$ background component by $\pm\sigma$ separately. In addition we shifted the background to the left and right side to model a large positive or negative asymmetry. The uncertainty due to the charge fake rate is estimated by reweighting the Monte Carlo sample according to the charge fake rate uncertainties. The uncertainty due to the uncertainty on the modeling of the number of z vertices is estimated by dividing the Monte Carlo sample into four subsamples with $n_{ZVtx} = 1$, $n_{ZVtx} = 2$, $n_{ZVtx} = 3$ and $n_{ZVtx} \geq 4$ and taking half of the maximal deviation as uncertainty.

In addition we checked the influence of different values for the top quark mass. In the region of interest $\pm 5\,\mathrm{GeV/c}^2$ around $175\,\mathrm{GeV/c}^2$, our method reproduces the real asymmetry within ± 0.01.

By adding all single uncertainties in quadrature we obtain a total systematic uncertainty of ± 0.051 for the inclusive measurement of the asymmetry.

IX. RESULTS

We have presented a measurement of the charge asymmetry in top pair production with an integrated luminosity of approximately 1.7 fb^{-1} collected with the CDF II detector. We measure the asymmetry in the lorentz invariant quantity $\Delta y \cdot Q_l$ defined as the rapidity difference of the semileptonically decaying top quark and hadronically decaying top quark multiplied with the charge of the lepton (e, μ).

Taking the systematic uncertainties into account, we measure an inclusive charge asymmetry of:

$$A^{\Delta y \cdot Q_l} = 0.28 \pm 0.13\,(\mathrm{stat.}) \pm 0.05(\mathrm{syst.})$$

which comes out higher than the expected asymmetry of $(4-7)\%$ predicted by NLO calculations but which is consistent with the NLO prediction within the errors.

To study the two different interference contributions to the inclusive asymmetry we measured the asymmetry as a function of the number of reconstructed jets. Within the statistical uncertainties our measurement is consistent with a prediction made by the NLO Monte Carlo MC@NLO. So far, there is no indication, that the high asymmetry value measured in the inclusive analysis comes from either the $t\bar{t}$ interference contribution or the $t\bar{t}g$ interference contribution alone.

Acknowledgments

We thank the Fermilab staff and the technical staffs of the participating institutions for their vital contributions. This work was supported by the U.S. Department of Energy and National Science Foundation; the Italian Istituto Nazionale di Fisica Nucleare; the Ministry of Education, Culture, Sports, Science and Technology of Japan; the Natural Sciences and Engineering Research Council of Canada; the National Science Council of the Republic of China; the Swiss National Science Foundation; the A.P. Sloan Foundation; the Bundesministerium für Bildung und Forschung, Germany; the Korean Science and Engineering Foundation and the Korean Research Foundation; the Science and Technology Facilities Council and the Royal Society, UK; the Institut National de Physique Nucleaire et Physique des Particules/CNRS; the Russian Foundation for Basic Research; the Comisión Interministerial de Ciencia y Tecnología, Spain; the European Community's Human Potential Programme; the Slovak R&D Agency; and the Academy of Finland.

[1] CDF Collaboration, F. Abe *et al.*, *Phys. Rev. Lett.* **74** (1995) 2626;
DØ Collaboration, S. Abachi *et al.*, *Phys. Rev. Lett.* **74** (1995) 2632.

[2] T.E.W. Group (Tevatron Electroweak Working Group) (2007), hepex//0703034.

[3] J.H. Kühn and G.Rodrigo, *Phys. Rev. Lett.* **81** (1998) 49.

[4] J. H. Kühn and G.Rodrigo, *Phys. Rev.* **D59** (1999) 054017.

[5] W.-M. Yao *et al.*, Journal of Physics G **33** (2006).

[6] S. Dittmaier, P. Uwer, S. Weinzerl, *NLO QCD corrections to ttbar+jet production at hadron colliders*, hep-ph/0703120.

[7] S. Frixione, P. Nason, and B.R. Webber, *Matching NLO QCD and parton showers in heavy flavour production*, hep-ph/0305252.

[8] D. Hirschbühl, *Measurement of the Charge Asymmetry and the W Boson Helicity in Top-Antitop Quark Events with the CDF II Experiment* (2005), IEKP-KA/2005-25.

[9] J. Weinelt, *Measurement of the charge asymmetry in top-antitop quark production with the CDF II experiment*, Fermilab-Masters-2006-05, Dec. 2006.

[10] DØ Collaboration, *A measurement of the forward-backward charge asymmetry in top pair production*, conference note 5393.

[11] T. A. Schwarz, *Measurement of the Front Back Asymmetry in Top-Antitop Quark Pairs Produced in Proton-Antiproton Collisions at Center of Mass Energy = 1.96 TeV*, PhD thesis, FERMILAB-THESIS-2006-51.

[12] D. Acosta *et al.*, Phys. Rev. D **71**(2005) 032001.

[13] S. Mrenna and P. Skands, PYTHIA *6.4 Physics and Manual*, hep-ph/0603175 T.Sjöstrand et al., Comput. Phys. Commun. **135**, 238 (2001).

[14] G. Marchesini, B.R. Webber, G. Abbiendi, I.G. Knowles, M.H. Seymor, and L.Stanco, *Computer Phys. Commun.* **67** (1992) 465.

[15] M. Mangano, ALPGEN, *A Generator for Hard Multiparton Processes in Hadronic Collisions*, JHEP 0307:001 (2003).

[16] A. Affolder *et al.* (CDF), Phys. Rev. D **71**(2001) 032002.